Transient buoyant convection in a 45°-inclined enclosure heated and cooled on adjacent walls

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Abstract

Purpose – To conduct a numerical study on two-dimensional, transient, buoyant flow inside an air-filled 45°-inclined enclosure, heated and cooled on adjacent walls.

Design/methodology/approach – The governing equations obtained through the stream function-vorticity formulation are solved using finite differences. Flow characteristics have been investigated for an aspect ratio of 1. Calculations are carried out for the Rayleigh numbers in the range of $10^3 \leq Ra \leq 5 \times 10^7$.

Findings – With the increasing Rayleigh number, four distinct flow regimes were identified based on the time variations of the mid-point stream function and the mean Nusselt number at the heated wall as well as those of flow and temperature fields: steady flow with symmetric two cells at low Ra; steady flow with asymmetric two cells at lower moderate Ra; oscillatory flow with the periodic nature at upper moderate Ra ; and oscillatory flow in chaotic nature at higher Ra range.

Originality/value – The distinct flow regimes are observed only at $\phi = 45^\circ$; a small deviation of the tilting angle from $\phi = 45^{\circ}$ results in the disappearance of the distinction.

Keywords Convection, Flow measurement, Rayleigh-Ritz methods

Paper type Research paper

Nomenclature

- $g =$ gravitational acceleration (m/s²)
- H = height of the enclosure (m)
 L = length of the enclosure (m)
- $=$ length of the enclosure (m)
- $n =$ coordinate in normal direction
- $Nu =$ Nusselt number
- Pr = Prandtl number
-
- Ra = Rayleigh number
 T = temperature (K) $=$ temperature (K)
- $t = \text{time (s)}$
- $u =$ velocity component in *x*-direction (m/s)
- $U =$ non-dimensional velocity component in x-direction
- $v =$ velocity component in y-direction (m/s)
- $V =$ non-dimensional velocity component in y-direction

 x, y = coordinates defined in Figure 1

 X, Y = non-dimensional coordinates

Greek symbols

- α = thermal diffusivity (m²/s)
- β = volumetric thermal expansion coefficient (1/K)
- ζ = non-dimensional vorticity
- θ = non-dimensional temperature
- τ = non-dimensional time
- Ψ = non-dimensional stream function
- Φ = generalized non-dimensional variable
- ϕ = inclination angle

Subscripts

$cr = critical$

 $C = \text{cold wall}$

Transient buoyant convection

451

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1. Introduction

Buoyancy-induced flow phenomenon in enclosures has been one of the most important topics in heat transfer research due to its importance in numerous engineering applications such as heating and cooling of buildings, cooling of electronic components, solar energy collection systems, materials processing and energy storage systems. Recent reviews of some studies on this topic have been given by Bejan (1984), Yang (1987), Ostrach (1988) and Raithby and Hollands (1998).

Despite the practical importance of transient effects in many enclosure-natural convection cases present in practice, accounts of this situation are relatively scarce when compared to the steady-state regime (Lage and Bejan, 1993; Fusegi and Hyun, 1994; Kwak and Hyun, 1996). For an enclosure exposed to a sudden vertical temperature difference, Patterson and Imberger (1980) presented a classical configuration for the transient process, which has then been examined by many other researchers (Yewell et al., 1982; Patterson, 1984; Ivey, 1984; Schladow et al., 1989; Paolucci and Chenoweth, 1989; Schladow, 1990; Paolucci, 1990; Patterson and Armfield, 1990; Armfield and Patterson, 1991; Jeevaraj and Patterson, 1992; Hyun and Lee, 1988; Wakatani, 1996; Kamakura and Ozoe, 1996; Cless and Prescott, 1996; Tagawa and Ozoe, 1996; Chung and Hyun, 1997; Nishimura *et al.*, 1997) taking into account of effects of some parameters such as Prandtl number, temperature-dependent viscosity, numerical schemes used, wall conduction, temperature-dependent density. Another scarcity of information in the existing literature is related to boundary conditions different to either a horizontally or a vertically imposed temperature difference, which are often expected to be encountered in practice (Poulikakos, 1985; November and Nansteel, 1987; Ganzarolli and Milanez, 1995). Aydin et al. (1999a) considered a two-dimensional rectangular enclosure heated from one side and cooled from above and investigated the effects of aspect ratio and Rayleigh number on the flow and heat transfer.

A numerical and experimental investigation into two-dimensional transient natural convection of single-phase fluids inside a completely filled square enclosure with one vertical wall cooled (subjected to a step change in temperature) and the other three walls insulated was conducted by Nicolette *et al.* (1985). In a similar numerical and theoretical study, Hall et al. (1988) considered transient natural convection heating of a two-dimensional rectangular enclosure filled with fluid. The heating was applied suddenly along one of the sidewalls, while the remaining three walls were maintained insulated. Upton and Watt (1997) carried out an experimental study on transient natural convection in an inclined rectangular enclosure, where the transient flow was initiated by heating and cooling of two opposing wails. A numerical investigation of transient laminar natural convection and the associated flow-mode transition in a differentially heated, inclined enclosure was carried out by Tzeng et al. (1997). In a recent article, Aydin (1999) investigated two-dimensional transient convection of single-phase fluids inside a square enclosure, which was driven by instantaneously raising and lowering the temperatures of the left side and the top walls, respectively.

In a recent study, Aydin et al. (1999b) examined laminar natural convection in an inclined square enclosure heated and cooled on adjacent walls. The effects of inclination angle and the Rayleigh number on the flow and temperature fields were investigated, and the critical inclination angles leading to the maximum and minimum heat transfer were determined. For $\phi = 45^{\circ}$ they observed intrinsic flow characteristics with increasing Ra , which is the motivation of this study. Here, at this critical inclination angle, we study the heat and flow characteristics with the increasing Rayleigh number.

2. Mathematical formulation and numerical method

The flow configuration of interest is shown in Figure 1. The square enclosure is of width and height H , and the Cartesian coordinates, x , y , with the corresponding velocity components (u, v) are indicated therein. Initially, the fluid is motionless and at a uniform temperature of T_o which is equal to $(T_H + T_C)/2$. Starting from time $t = 0$, the temperature of the left lower wall is raised to $T_H = T_o + \Delta T$, while the temperature of the upper left wall is lowered to $T_c = T_o - \Delta T$ simultaneously. These temperatures are maintained thereafter. An adiabatic type of boundary condition is used with the remaining walls. The fluid is assumed to be incompressible, with constant properties, although buoyancy effects are considered by invoking the Boussinesq approximation. Through the usual vorticity-stream function formulation under the Boussinesq approximation, the following set of non-dimensional equations are obtained:

$$
\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\zeta \tag{1}
$$

$$
\frac{\partial \zeta}{\partial \tau} + U \frac{\partial \zeta}{\partial X} + V \frac{\partial \zeta}{\partial Y} = Pr \left(\frac{\partial^2 \zeta}{\partial X^2} + \frac{\partial^2 \zeta}{\partial Y^2} \right) + Ra \, Pr \left(\frac{\partial \theta}{\partial X} \cos \phi - \frac{\partial \theta}{\partial Y} \sin \phi \right) \tag{2}
$$

$$
\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}
$$
(3)

where dimensionless variables and the dimensionless forms of the stream function and the vorticity are defined as the following:

$$
X = \frac{x}{H} \quad Y = \frac{y}{H} \quad \theta = \frac{T - T_C}{T_H - T_C} \quad \tau = \frac{\alpha t}{H^2} \quad U = \frac{u}{\alpha/H} \quad V = \frac{v}{\alpha/H} \tag{4}
$$

$$
U = \frac{\partial \Psi}{\partial Y} \quad Y = -\frac{\partial \Psi}{\partial X} \quad \zeta = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}
$$
(5)

In the above equations, Pr , is the Prandtl number and Ra is the Rayleigh number defined in the following forms:

$$
Pr = \frac{\nu}{\alpha} \quad Ra = \frac{g\beta H^3 (T_H - T_C)}{\nu \alpha} \tag{6}
$$

In accordance with the problem statement, the initial conditions are:

Transient buoyant convection

$$
U = 0 \quad V = 0 \quad \theta = \frac{1}{2} \quad \text{at } \tau = 0 \tag{7}
$$

which imply zero velocity and uniform temperature. The appropriate boundary conditions are:

$$
\theta = 1 \quad U = 0 \quad V = 0 \text{ at } X = 0 \text{ in } 0 < Y < 1 \tag{8}
$$

$$
\frac{\partial \theta}{\partial X} = 0 \quad U = 0 \quad V = 0 \quad \text{at } X = 1 \text{ in } 0 < Y < 1 \tag{9}
$$

$$
\frac{\partial \theta}{\partial Y} = 0 \quad U = 0 \quad V = 0 \quad \text{at} \quad Y = 0 \quad \text{in} \quad 0 < X < 1 \tag{10}
$$

$$
\theta = 0 \quad U = 0 \quad V = 0 \text{ at } Y = 1 \text{ in } 0 < X < 1 \tag{11}
$$

Imposition of the condition of the no-slip along the solid boundaries of the enclosure on the definition of the stream function yields the condition of $\Psi_{wall} = constant$. Thus, at the boundaries of the enclosure, Ψ is arbitrarily specified as zero. The value of the vorticity at the boundary, Ψ_{wall} is calculated from:

$$
\zeta_{\text{wall}} = -\frac{\partial^2 \Psi}{\partial n^2} \tag{12}
$$

where n is the outward drawn normal of the surface. In numerical calculations, the values of vorticity at corners are taken as averages of the values of vorticity at two neighboring nodes. The treatment of the temperature of the corner of neighboring hot and cold walls can be found in Aydin et al. (1999a).

The numerical solutions to the systems of coupled differential equations given above are obtained using the finite difference method described in detail in Patankar (1980). The vorticity transport and energy equations are solved using the alternating direction implicit method of Peaceman and Rachford (Roache, 1982), and the stream function equation is solved by SOR (successive over-relaxation) method (Patankar, 1980). Details of the numerical simulation method and its validity can be found in Aydin et al. (1999a, b).

The solution domain is discretized using a non-uniform mesh with smaller grid spacings near the walls and larger spacings in the interior, which allows the hydrodynamic and thermal boundary layers to be resolved without an excessive number of nodes. As a compromise between cost and accuracy, all the computations presented here are performed on a 41×41 grid, which is verified to lead grid-independent solution by performing solutions on finer meshes (e.g. 61×61 , 81×81). Time step chosen was 0.0005.

Once the temperature distribution is obtained through the above numerical procedure, the local Nusselt number, $Nu(Y)$, is calculated from:

$$
Nu(Y) = \left[-\frac{\partial \theta}{\partial X} \right]_{X=0} \tag{13}
$$

based on which the average Nusselt number, \overline{Nu}_{v} for the heated wall is determined through the below equation:

$$
\overline{Nu_y} = \int_0^1 Nu(Y) dY \tag{14}
$$

3. Results and discussion

The main purpose of this study is directed toward the predictions of the transient characteristics of fluid flow and energy transport in a 45° -inclined enclosure heated and cooled from adjacent walls. The inclination angle is fixed at $\phi = 45^{\circ}$ because the interesting features of flow field at this value are not observed for the neighboring inclination angles (Aydin *et al.*, 1999b). Air with $Pr = 0.71$ was considered as a working fluid and the dependence of the flow and temperature fields on Rayleigh number is determined for $10^3 \leq Ra \leq 5 \times 10^7$. Initially, the fluid is at a uniform temperature and motionless. A sudden differential heating at $t = 0$ on the left-hand side adjacent walls is employed. The lower wall is kept at a uniform temperature of T_H and the upper one is kept at a uniform temperature of $T_c \ll T_H$). Then, transient evolution of flow and temperature fields are determined numerically.

Numerical calculations revealed that the flow pattern exhibits a very strong dependence on the Rayleigh number, and several different flow regimes exist within the enclosure depending on the Rayleigh number. On the basis of the different flow structures, we identify four different flow regimes. In order to determine the critical

Transient buoyant convection

HFF 16,4

456

Rayleigh number at which the transition from one regime to another occurs, a kind of interval halving procedure is applied.

As explained earlier, in order to make sure that the oscillatory behavior of the flow is not the one with numerical character, numerical solutions were checked for various grid sizes and time steps and it was found that the grid size and time step show a minor effect on the character of the oscillations. In the following, four distinct flow regimes and their characteristics are defined.

For $Ra < 8 \times 10^4$, steady-state solutions are reached after a short period of time. However, after the solutions are settled down the steady state, the flow fields observed are classified into two regimes depending on the geometrical structure of the flow cells: axisymmetric and asymmetric. For $Ra \leq 3.375 \times 10^3$, the overall structure is axisymmetric about the horizontally oriented diagonal of the inclined enclosure with the two counter-rotating cells. As a typical example for this region, Figure 2 shows the streamline and isotherm patterns for $Ra = 10³$. An instantaneous temperature rise at the lower left wall accelerates a fluid pack next to it. Rising along the hot wall, this fluid pack forms a clockwise rotating cell and interacts with a counter-clockwise rotating upper cell which is formed due to the simultaneous temperature drop at the upper left wall. The two symmetric cells are so weak that the corresponding isotherms show a diagonally symmetric distribution, which is an indication of the fact that conduction is the dominant mechanism of heat transfer for this case. For $3.375 \times 10^3 < Ra < 8 \times 10^4$. the symmetry of the two cells about the horizontal diagonal of the enclosure is distorted because of considerable effect of convection on energy transfer. With increasing Ra in this region, the lower cell tends to occupy all the space inside the enclosure. For $Ra = 2 \times 10^4$, all the space inside the enclosure is invaded by the lower cell restricting the upper cell to a comparably negligible area in the upper corner (Figure 3). For $Ra \geq 8 \times 10^4$, steady-state never reached. The flow exhibits an oscillatory character in this region. The oscillations are periodic and have small magnitudes at lower values of the Rayleigh number. At $Ra = 5 \times 10^7$, oscillations are not periodic any longer, suggesting a chaotic-like behavior. The oscillatory motion stems from the growth and reduction in size of the two main cells interchangeably. To better explain the phenomenon, Figures 4 and 5 are drawn, which shows the variations of Ψ_{mid} and \overline{Nu} with time for various Ra , respectively. As it can be seen from these figure, after a short

Figure 2. Streamlines and isotherms for $Ra = 10^3$ (symmetric steady regime)

period of time, the transient records of both Ψ_{mid} and \overline{Nu} assume an oscillatory character. As the Ra increases, the amplitudes of the oscillations become larger, while their periods become shorter. On the basis of the Ra dependence of the transient behavior of $\Psi_{\rm mid}$, two characteristic regime classifications can be made: periodic regime at lower Ra and chaotic regime at higher Ra. The transition from the periodic regime to the chaotic regime occurs around $Ra = 1.12 \times 10^7$. $Ra = 10^5$ and $Ra = 5 \times 10^7$ are typical examples for periodic and chaotic regimes, respectively. In fact, the periodic regime is in a sinusoidal character at some intermediate Rayleigh numbers (for example, at $Ra = 10^6$). If the time variations of the Ψ_{mid} for various Ra are carefully examined, it will be recognized that the mean value of Ψ_{mid} increases with increasing Ra. This means that the lower cell having positive (clockwise direction) circulation will be enhanced with the increasing Rayleigh number. In fact, as expected, a variation of Ψ_{mid} about a mean value of zero implies that the upper and bottom cells have the same extreme dimensions; i.e. their maximum and minimum sizes are identical. In order to have a better view on the behaviors explained above, Figures 4 and 5 are reproduced in a narrow range of the dimensionless time interval from 1 to 1.2. For this range, Figures 6 and 7 show the variations of Ψ_{mid} and Nu with time for various Ra, respectively.

Some further information about the spatial structure of this oscillatory behavior can be best obtained by looking at the evolution of the flow and temperature fields within time. As an example case, Figure 8(a) shows the instantaneous streamlines and isotherms at the times 1, 2, ... and 5 as marked in Figure 8(b), during one period of oscillation for $Ra = 10^5$. It can be clearly seen that the fluid oscillates between two extreme positions of the upper and lower cells. The two cells undergo the same oscillation process, but with a 180° phase shift. The two cells compete against each other, each becoming alternately small and large. Each cell, one is located at the lower half and the other is located at the upper half, reaches its maximum size, consequently constraining the other cell to be of minimum size. This phenomenon can be attributed to the momentum diffusion between the two counter-rotating cells. As pointed out by Aydin et al. (1999b), destabilizing effects of the two isothermal walls contribute equally to the periodic nature of the flow within the enclosure at this specific orientation ($\phi = 45^{\circ}$). A small deviation from $\phi = 45^{\circ}$, whether it is positive or negative, results in destruction of this balance and the effect of one isothermal wall becomes dominant. To better explain this phenomenon, two neighboring orientations, one with $\phi = 40^{\circ}$ and

(continued)

 $Ra = 5x10^{7}$

Figure 4.

Figure 5. Variation of \overline{Nu} with τ

(continued)

 $Ra = 5x10^{7}$

Figure 5.

range of τ from 1 to 1.2

the other with $\phi = 50^{\circ}$, are considered and the corresponding flow and temperature data are shown in Figure 9. At $\phi = 40^{\circ}$, the effect of the hot wall becomes dominant and the lower cell occupies more space restricting the upper cell to a smaller space. At $\phi = 50^{\circ}$, on the other hand, the effect of the cold wall becomes dominant and the relative sizes of the two cells turn other way around as one can observe from Figure 9. In fact, these two opposite features found in two neighboring tilting angle values explain the interesting scenery existing at intermediate tilting angle value, $\phi = 45^{\circ}$.

As far as the heat transfer within the enclosure is concerned, the main quantity of physical interest is the normalized average Nusselt number, whose value at the heated wall as a function of Rayleigh number is plotted in Figure 10. At low Ra range $Ra < 8 \times 10^4$, the flow is in a steady-state character with two non-mixing cells, whether they are symmetric or asymmetric, and the effect of Ra number on Nu is not significant since the dominant heat transfer mechanism in this case is diffusion. At moderate Ra range $(8 \times 10^4 \leq Ra \leq 1.12 \times 10^7)$, the energy transport is enhanced considerably due to the periodic structure of the flow field and a significant variation of Nu with Ra occurs in this range. Beyond $Ra = 1.12 \times 10^7$, the effect of Ra on Nu becomes more profound due to the transition to the chaotic flow regime. In both periodic and chaotic regimes, a better mixing occurs when compared with the steady-state regimes observed at low Ra range.

4. Conclusions

A numerical study is conducted on transient, laminar, buoyant flow inside an air-filled and 45^o-inclined square enclosure. Flow is initiated by instantaneously triggering the temperatures of the left-hand side adjacent walls; increasing the lower wall temperature and decreasing the upper wall temperature. The development of the flow and temperature fields following these step changes in wall temperatures are determined

numerically. On the basis of numerical data of the flow field, four distinct flow regimes are identified and the critical values of Ra at which the transition from one regime to another occurs, are determined:

- (1) For $Ra \leq 3.375 \times 10^3$, a steady-state regime with two symmetric cells is established.
- (2) For the range of $3.375 \times 10^3 < Ra < 8 \times 10^4$, again a steady-state regime but with asymmetric two cells is established.
- (3) For the range of $8 \times 10^4 \leq Ra < 1.12 \times 10^7$, an oscillatory regime in a periodic nature is distinguished.
- (4) For $Ra \geq 1.12 \times 10^7$, an oscillatory regime in a chaotic nature is identified. Interestingly, these distinct flow regimes are observed only at $\phi = 45^{\circ}$, a small deviation of the tilting angle from $\phi = 45^{\circ}$ results in the disappearance of the distinction.

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